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Cooperative Task Assignment/Path Planning of Multiple Unmanned Aerial Vehicles Using Genetic Algorithms

Yeonju Eun* and Hyochoong Bang[†] Korea Advanced Institute of Science and Technology, Daejon 305-701, Republic of Korea

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I. Introduction

C OOPERATIVE control of multiple UAVs (unmanned aerial vehicles) and/or (unmanned combat aerial vehicles) has been an emerging issue for future application to sophisticated military missions. In particular, various new concepts using UAVs for challenging missions are under active investigation. In the scenarios of some special military missions such as Wide Area Search and Destroy, Intelligence Surveillance and Reconnaissance, and Suppression of Enemy Air Defense (SEAD), the team cooperation of multiple UAVs is key to accomplishing these missions; to effectively use multiple UAVs, task assignment and path planning are crucial steps taken before an actual operation. In this paper, we discuss cooperative task assignment and path planning of multiple UAVs for SEAD missions.

The main purpose of a SEAD mission is to attack ground targets using multiple UAVs. Further constraints can be added to this problem, depending on circumstances. For example, avoiding the obstacles and threats with simultaneous arrivals at the target location, or sequential tasks (such as detection, destruction, and verification on one target) on each target, may be required. In this work, one of the typical mission scenarios of the SEAD mission using multiple UAVs is used. It is the cooperative attack on ground targets. In the mission scenario considered in this paper, the primary terrain information, such as locations of targets, obstacles, and dangerous areas are assumed to be already known. Because the targets we are interested in are either fixed on the ground or moving very slowly, it can be assumed to be stationary. In addition to these circumstances, more crucial constraint is given to increase the probability of mission success. More than one aircraft must be assigned to each target, and the UAVs assigned to the same target should make rendezvous at the target location for simultaneous attack or should arrive sequentially

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to accomplish consecutive multiple tasks. In this paper, said constraint for the rendezvous or sequential arrivals at the target location is called the timing constraint.

The task assignment problem in the different types of SEAD mission scenarios is addressed in many papers [1–3]. In that scenario, it is assumed that the targets are moving on the ground, and as the mission progresses, the target location information is updated and shared with every member in the team. In this type of mission, the "cooperative ground moving target engagement" is the main problem. However, in these earlier works, the flight restricted zones due to obstacles or threats are not considered at all, and the timing constraints are not given either. In [4], routing heterogeneous multiple UAVs, which have different roles in a mission, is solved by using mixed integer linear programming. A similar concept to our work was used in using waypoints and network shape of candidate paths, but simultaneous attacks or multiple consecutive tasks with time delays were not considered.

The main contribution of our work is that the timing constraints for simultaneous attacks and multiple consecutive tasks with specified time delays were applied for more advanced types of missions. Moreover, task assignment and path planning problems for a group mission of multiple UAVs are solved concurrently.

The paper's subsequent sections are organized as follows: Sec. II provides the primary strategy before using the genetic algorithm (GA), Sec. III gives the genetic algorithm formulation, Sec. IV shows simulation results, and Sec. V presents the final conclusions.

II. Preparation for Genetic Algorithm

A. Problem Description

In this study's scenario, it is assumed that the primary information of the terrain is initially given. We know the location of targets, physical obstacles, and threats. There are N_T targets and N_U UAVs starting from different locations. Each team, assigned ith target, has to be made up of n_i UAVs. The number of UAVs in each team n_i is predetermined. Except for the team with a single member, all members in each team have to make a rendezvous at the target location for simultaneous attack, or arrive sequentially to achieve consecutive multiple tasks, such as classification of target, attack, and verification of destruction. Therefore, a constraint, named here as timing constraints, for rendezvous or sequential arrival is required, and this constraint originates from the available speed range of the UAVs, $V_{\min} \leq V \leq V_{\max}$. In the case of consecutive multiple tasks, the time intervals between tasks are also predetermined. The available speed range of UAVs can differ, but mission capabilities, such as capability of detection or destruction, are all the same, meaning that every UAV is equipped with the same detection sensors or munitions, and there is no UAV that must be assigned to a specific mission.

B. Candidate Paths for Genetic Algorithm

To satisfy the timing constraints, not only is the optimal path needed, but several candidate paths from each UAV to each target are also needed. There are many methods for path planning with obstacle avoidance, but most of them cannot determine multiple paths. A few useful methods of finding multiple paths have been proposed in previous papers. In [3], the potential field method is used to find multiple candidate paths for timing constraint in a SEAD mission. Hence, in general, the multiple paths found by using this potential

^{*}Ph.D. Candidate, Department of Aerospace Engineering, School of Mechanical, Aerospace and Systems Engineering; yjeun@ascl.kaist.ac.kr. Student Member AIAA.

[†]Associate Professor, Department of Aerospace Engineering, School of Mechanical, Aerospace and Systems Engineering; hcbang@ascl.kaist.ac.kr. Senior Member AIAA.

method have similar lengths. It is obvious that various path lengths are more conducive to finding feasible assignment that satisfy the timing constraints. As mentioned in [3], because the Voronoi diagram generates paths in the shape of a net, we can find many candidate paths with greater length variety. Voronoi diagrams are used to find multiple candidate paths while avoiding regions with dangerous threats, such as radars, in [5-7]. However, as it is only a conventional Voronoi diagram, it is not appropriate for finding paths to avoid physical obstacles. On the other hand, a visibility graph method can design paths to avoid physical obstacles properly, and can also provide multiple candidate paths. But the visibility graph method is not proper for finding the path to avoid threats that have no specific shape. To make up for the weaknesses of each method, hybrid algorithms of visibility graphs and the Voronoi diagram are proposed in [8,9]. Essentially, multiple candidate paths of various lengths are needed for the GA formulation. Because Voronoi diagrams or hybrid algorithms provide the paths in the shape of a net, multiple candidate paths of various lengths can be easily found by using the algorithm for the k-shortest path-finding problem of [10].

III. Genetic Algorithm Formulations

The main reason GA is used in this problem is that it is not restricted to continuity, differentiability, or unimodality of searching domain for the optimal solution; GA is also a very powerful method for solving not only an optimization of a polynomial cost function, but also a combinatorial optimization problem. In this chapter, a grouping and assignment problem of multiple UAVs is formulated as a combinatorial optimization problem.

A. Chromosome Representation

To simplify this problem, let us assume that the total number of available UAVs is equal to the total number of tasks. Table 1 gives an example of a solution expressed as a chromosome. In this situation, there are three targets, and six UAVs are available. Each group, made up of two UAVs, has to be assigned to each target.

The chromosome represented as Table 1 indicates that UAV1 is assigned to target 1, and the second path of UAV1 is selected among N_P candidate paths. Similarly, UAV3 is assigned to target 2 and the first path is selected. The first team, which is assigned to target 1, has two members, UAV1 and UAV2. The second team, UAV3 and UAV6, are assigned to target 2, and the third team, assigned to target 3, consists of UAV4 and UAV5. The chromosome representations in similar form are used in [1,2]. However, the circumstances and objectives of the mission are different from our work, and, naturally, the problem formulation is also different. To solve the problem formulated in our work, we modified the representation of [1,2].

Table 1 Chromosome representation example

UAV no.	UAV1	UAV2	UAV3	UAV4	UAV5	UAV6
Target no. Path no.	1 2	1 4	2 1	3 1	3 3	2 6

Table 2 Chromosome representation example

UAV no.	UAV1	UAV2	UAV3	UAV5	UAV6
Target no.	1	3	3	2	2
Path no.	2	4	1	6	3

Let us consider the chromosome representation when the number of available UAVs in the mission is not equal to the total number of tasks. In the case represented by Table 2, we have six available UAVs, but the total number of tasks is five. The chromosome of Table 2 means UAV4 is selected at random to be excluded from the mission.

If the total number of available UAVs is smaller than the total number of tasks, some UAVs must be assigned to more than two targets; such a scenario is named a "double assignment" in our work. The chromosome of Table 3 shows that UAV2 is assigned to target 3 and target 2, and should go to the location of target 2 after completing a task at target 3. The UAVs that are assigned to more than two targets are selected at random. According to this example, the UAV2 will use the first candidate path among N_P candidate paths, which were founded to go to target 2 from target 3.

B. Constraints

To satisfy these requirements and maximize the teams' abilities to accomplish their entire mission, there are two constraints which must be considered.

1. Timing Constraint

For simultaneous attacks or sequential tasks on each target, a timing constraint is needed. Once grouping has been completed and a path from each UAV to each target has been selected, the required time for arrival can be estimated. After the target is assigned and the path selected, each UAV's time over target (TOT) [11] can be computed, and we can select best TOT for each team. Because this assignment and path planning are at the highest level of mission operation, the TOT can be simply calculated as Eq. (1):

$$\min \text{TOT}_{k,i} = L_{k,i}/V_{k,\text{max}} \qquad \max \text{TOT}_{k,i} = L_{k,i}/V_{k,\text{min}} \qquad (1)$$

where the subscript k denotes the identification number of UAV(k), and i denotes the target(i) or the ith team to attack the target(i). So, $L_{k,i}$ is the length of the selected path of UAV(k) assigned to target(i), and the subscript k should satisfy $k \in G_i$, and $G_i = \{k | k \text{ is identification (ID) number of UAV that is a member of <math>i$ th team} = $\{g_1, g_2, \ldots, g_{n_i}\}$. In case of rendezvous, the best TOT of each team, TOT^* , can be determined as Eqs. (2–4):

$$TOT_i^* = \min t, \qquad t \in TOT_i \tag{2}$$

$$TOT_i = TOT_{g_1,i} \cap TOT_{g_2,i} \cap \dots \cap TOT_{g_n,i}$$
 (3)

$$TOT_{k,i} = \{t | \min TOT_{k,i} \le t \le \max TOT_{k,i} \}$$
 (4)

Therefore, the timing constraint can be given as though the TOT* determined by Eqs. (2–4) must exist.

When applying the timing constraint to multiple consecutive tasks, it is possible to make the timing constraints more general. For this case, let $\Delta t_{n-1 \to n}$ be the time interval between the (n-1)th task and nth task on same target. Let G_i satisfy the condition given in Eq. (5):

$$\min \text{TOT}_{g_1,i} \leq \min \text{TOT}_{g_2,i} \leq \dots \leq \min \text{TOT}_{g_{n_i},i}$$

$$(g_k \in G_i, k = 1, 2, \dots, n_i)$$
(5)

Then, we can determine the set of TOT' by using Eq. (6):

Table 3 Chromosome representation example

UAV no.	UAV1	UAV2	UAV3	UAV4	UAV5	UAV6	UAV2	UAV3	UAV5
Target no.	1	3	3	2	1	3	2	1	2
Path no.	2	4	1	6	5	4	1	3	3

Table 4 Example of CX: modification of parents

						P	arent	1						
UAV no.	1	2	3	4	5	6		UAV no.	1	2	3	4	5	6
Target no.	2	2	3	3	1	1	\rightarrow	Target no.	2	5	3	6	1	4
Path no.	2	4	1	1	3	5		Path no.	2	4	1	1	3	5
						P	arent	2						
UAV no.	1	2	3	4	5	6		UAV no.	1	2	3	4	5	6
Target no.	1	2	1	3	2	3	\rightarrow	Target no.	1	2	4	3	5	6
Path no.	1	2	1	4	2	3		Path no.	1	2	1	4	2	3

Table 5 Example of CX: offspring chromosomes

UAV no.	1	2	3	4	5	6	1	2	3	4	5	6
Target no.	2	5	4	3	1	6	1	2	3	6	5	4
Path no.	2	4	1	4	3	3	1	2	1	1	2	5

$$\begin{aligned} \operatorname{TOT}_{g_{2},i}' &= \{t | \min \operatorname{TOT}_{g_{2},i} - (\Delta t_{1 \to 2}) \leq t \leq \max \operatorname{TOT}_{g_{2},i} \} \\ \operatorname{TOT}_{g_{3},i}' &= \{t | \min \operatorname{TOT}_{g_{3},i} - (\Delta t_{1 \to 2} + \Delta t_{2 \to 3}) \leq t \\ &\leq \max \operatorname{TOT}_{g_{3},i} \} \\ &\vdots \\ \operatorname{TOT}_{g_{n_{i}},i}' &= \{t | \min \operatorname{TOT}_{g_{n_{i}},i} - (\Delta t_{1 \to 2} + \Delta t_{2 \to 3} + \dots + \Delta t_{(n_{i}-1) \to n_{i}}) \\ &\leq t \leq \max \operatorname{TOT}_{g_{n_{i}},i} \} \end{aligned}$$

$$(6)$$

The best TOT, TOT*, can be determined by Eq. (7):

$$TOT_{i}^{*} = \min t, \qquad t \in TOT_{i}'$$

$$TOT_{i}' = TOT_{g_{1},i}' \cap TOT_{g_{2},i}' \cap \cdots \cap TOT_{g_{n},i}'$$
(7)

Therefore, the timing constraint can be given as that the TOT^* determined by Eq. (7) must exist. One more condition, which is given in Eq. (8), is needed for consecutive multiple tasks:

$$TOT_{i}^{*} + \Delta t_{1 \to 2} \in TOT_{2,i}$$

$$TOT_{i}^{*} + \Delta t_{1 \to 2} + \Delta t_{2 \to 3} \in TOT_{3,i}$$

$$\vdots$$

$$TOT_{i}^{*} + \Delta t_{1 \to 2} + \Delta t_{2 \to 3} + \dots + \Delta t_{(n_{i}-1) \to n_{i}} \in TOT_{n_{i},i}$$
(8)

Table 9 Mutation example

UAV no.	1	2	3	4	5	6		UAV no.	1	2	3	4	5	6
Target no. Path no.				3 4			\rightarrow	Target no. Path no.					1 3	

Table 10 Inversion example

UAV no.	1	2	3	4	5	6	UAV no.	1	2	3	4	5	6
_	2 2						Target no. Path no.	2 2					

2. Path Constraint

The purpose of a simultaneous attack is to disturb the target's defense system and improve the success rate of destruction. For this purpose, more than two UAVs must arrive at a target location simultaneously, and the UAVs should not follow the same approach directions. This constraint will be called a "path constraint" in our study, but a path constraint is applied to only a rendezvous case.

C. Genetic Operators

Reproduction, crossover, mutation, inversion, and elitism are used as genetic operators. In GA operations, a set of an assigned target number and a selected path number of each UAV acts as a gene, and the cost J of this problem is determined to be the sum time taken to complete all tasks at all target locations:

$$J = \sum_{i=1}^{N_T} T_i = \sum_{i=1}^{N_T} \left(\text{TOT}_i^* + \sum_{j=1}^{n_i - 1} \Delta t_{j \to j+1} \right)$$
(9)

where T_i is the required time to finish the tasks at the ith target. In case of a simultaneous attack, T_i is the required time to finish only one attack, but in cases where consecutive tasks must be completed, T_i is the required time to finish the final task at the location of the ith target.

1. Crossover

Because this is formulated as a kind of combinatorial optimization problem, an appropriate crossover operator should be used instead of a conventional one. To preserve the validity of the assignment, the crossover operators of a traveling salesman problem (TSP) can be used. In GA for TSP, partially matched crossover (PMX), order

Table 6 Example of CX (double assignment case): parent chromosomes

UAV no.	1	2	3	4	5	6	2	3	5	1	2	3	4	5	6	1	2	6
Target no.	1	3	2	6	4	9	5	7	8	2	1	5	8	4	7	3	6	9
Path no.	2	4	1	6	5	4	1	3	3	5	4	2	3	1	2	1	2	1

Table 7 Example of CX (double assignment case): offspring chromosomes

UAV no.	1	2	3	4	5	6	2	3	5	1	2	3	4	5	6	2	3	5
Target no.	1	3	2	8	4	7	5	6	9	2	1	5	6	4	9	3	7	8
Path no.	2	1	1	3	1	2	1	2	1	5	4	2	6	5	4	1	3	3

Table 8 Chromosome modification after crossover

UAV no.	1	2	3	4	5	6	2	3	5		1	2	3	4	5	6	1	3	5
Target no. Path no.						1 2				\rightarrow	1 2							3 2	

Table 11 Chromosome modification for inversion (double assignment case)

UAV no.	1	2	3	4	5	6	1	3	5		1	2	3	3	4	4	5	6
Target no. Path no.						1			2	\rightarrow			2					

Table 12 Example of inversion (double assignment case)

UAV no.		1	2		3	4		5	6			1	2	3		4		5	6
Target no.	1	3	3	2	3	2	1	2	1	\rightarrow	1	3	3	2	2	3	1	2	1
Path no.	2	4	5	1	2	3	1	1	2		2	4	5	3	3	2	1	1	2

Table 13 Optimal solution for case 1

UAV no.	UAV1	UAV2	UAV3	UAV4	UAV5	UAV6
Target no.	3	1	1	3	2	2
Path no.	6	4	2	1	2	3

crossover (OX), and cycle crossover (CX) can be used instead of a conventional crossover operator [12]. Instead of symbols for the cities in TSP, target numbers are used. However, it still seems incompatible to use the crossover operator of TSP because the target numbers are repeated in one chromosome. To overcome this problem, target numbers are changed, as we can see in Table 4, where the new ID number of the target in the ith gene $k_{i,\text{new}}$ is determined in Eq. (10):

$$k_{i,\text{new}} = k + (m-1)N_T$$
 (10)

where k is the original target number and m is the number of k which exist in front genes. After this modification, we can use PMX, OX, or CX. In our work, CX is used as the crossover operator. We can see how CX works in Table 5.

In the case where the total number of available UAVs is not equal to the total number of tasks, the crossover operator works as we can see in Tables 6 and 7. The ID numbers of UAVs and the first gene are transmitted to the same offspring chromosome. After crossover, the feasibility of each team formation must be checked, as the same UAV cannot exist in one team. For example, if one of the offspring chromosomes is given as shown in the left side of Table 8, the UAV2 is assigned to target 3 twice, which is not a feasible formation. Therefore, the UAV2 is changed into a randomly selected UAV, which is not assigned to target 3 as we can see in the right side of Table 8.

2. Mutation

The basic rules of mutation operation are the same with a simple genetic algorithm. In our work, however, only the path number of a selected gene can be changed at random, as shown in Table 9.

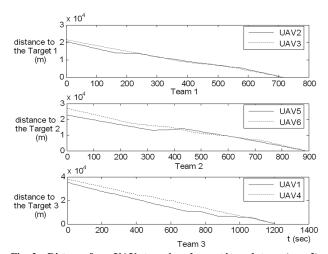


Fig. 2 Distance from UAVs to assigned target in each team (case 2).

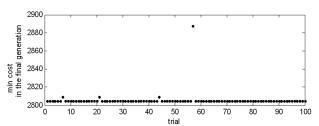


Fig. 3 Minimum cost in the final population of each run.

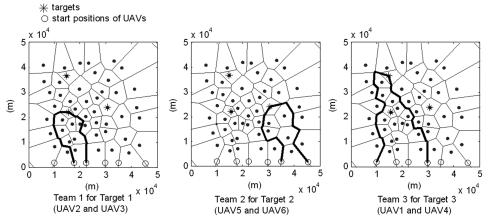


Fig. 1 Assignment results of the optimal solution (case 1).

Table 14 Optimal solution for case 2

UAV no.	UAV1	UAV2	UAV3	UAV4	UAV5	UAV6	UAV3	UAV5	UAV6
Target no.	3	1	1	3	2	2	2	1	3
Path no.	6	5	6	1	2	3	4	1	2

3. Inversion

Only the path number is changed through the mutation, even though the purpose of mutation is to maintain the variety of individuals despite the law of the survival of the fittest. So, we use the inversion operator to change the target assignment. In the inversion operation, two inversion cuts must be selected at random. For example, if the first inversion cut is located between the second and third gene, and the second inversion cut is located behind the sixth gene in the chromosome of the left side of Table 7, the resulting chromosome is changed, as shown in the right side of Table 10, by the inversion operation.

If the total number of available UAVs in the mission is smaller than the total number of tasks, the inversion operator works as follows. The chromosome representation is changed as we can see in Table 11. The number of genes is six in the chromosome of the right side of Table 11. Then, if the first inversion cut is located between the second and third gene, and the second inversion cut is located between the fourth and fifth gene, the chromosome is changed into the right side of Table 12 by inversion.

Instead of the aforementioned inversion operator, an insertion mutation or a reciprocal mutation can be used. However, a simple mutation is still required because a set of a selected path number and a target number will never be changed without a simple mutation.

D. Repair Algorithm

After the optimal solution is found, the timing constraint can be satisfied by using a delayed departure of UAVs, except for the UAV which will arrive last. The departure delay time can be obtained by using the TOT range sets given in Eqs. (4–8). However, in cases where the UAVs must depart at one time, or where a path constraint is required, a repair algorithm might be needed. The repair algorithm is applied on the assumption that the team formations are feasible, and so only path numbers can be changed by the repair algorithm. The path number of the UAV that can arrive at the target first will be changed first because the TOT_i^* of the ith team, which is determined by the UAV which has the maximum min $TOT_{g_k,i}$.

IV. Simulations

The following section presents a simulation study for two particular sample SEAD scenarios. For the six UAVs departing from different sites, the mission conditions include three targets and 32

threats whose positions are initially given. For each UAV, the available speed range is given as $(25 \le V \le 40 \text{ m/s})$.

A. Case 1

For case 1, two-member teams are assigned to simultaneously attack three targets, with a timing constraint and path constraint used for simultaneous attack. The optimal solution is shown in Table 13.

Figure 1 shows the trajectory tracking results, using the optimal assignment of Table 13. In trajectory generation, the nonlinear model predictive control method is used. We can see in Fig. 2 that the rendezvous mission of each team is achieved successfully.

A Monte Carlo simulation, consisting of 100 runs, is used to study the convergence performance. In each run, the maximum number of iterations (equal to the maximum number of generations) is limited to 1000 times. As we can see in Fig. 3, there are four cases in which the GA conversed to local minimum of failure in finding the optimal solution during 100 timed runs.

B. Case 2

In the second case, three targets are given to each team of three members. Simultaneous attacks by two UAVs and verification of destruction by one UAV are given as consecutive multiple tasks on each target. However, the number of available UAVs is still six; therefore, this case needs a double assignment. The time delay between the two consecutive tasks is given as 500 s. The optimal solution is shown in Table 14. Figures 4 and 5 show the trajectory tracking results of UAV5, UAV3, and UAV6, which are all assigned to more than two targets. UAV5 is assigned to the second team for simultaneous attack on target 2, and also to the first team for destruction verification of target 1. Each UAV assigned to verify destruction arrives at the target location 500 s after the other team members' simultaneous attack. This case, also a Monte Carlo simulation, consists of 100 runs, and is used to study the convergence performance. The convergence results of the Monte Carlo simulation are given in Fig. 6. The GA is converged to a local minimum in 14 cases, as seen in Fig. 6.

V. Conclusions

A grouping and assignment problem of a SEAD mission using multiple UAVs is formulated as a new combinatorial optimization problem, and the GA is used to solve. According to the scenario in this study, the multiple fixed targets are located on the ground, with

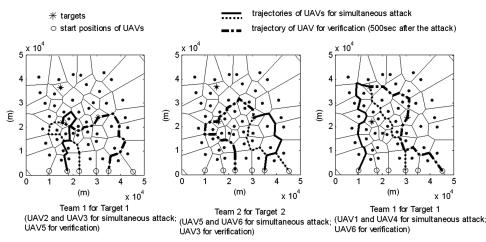


Fig. 4 Assignment results of the optimal solution (case 2).

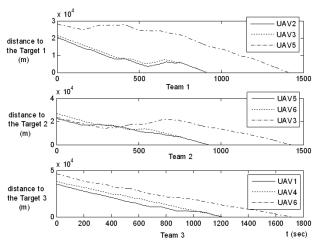


Fig. 5 Distance from UAVs to assigned target in each team (case 2).

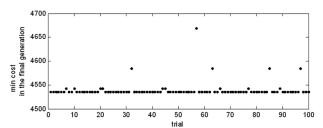


Fig. 6 Minimum cost in the final population of each run.

target locations initially given. To avoid threats and obstacles, a Voronoi diagram is used and multiple candidate paths are generated for the optimization with the GA. For various conditions of the mission, timing constraints and path constraints are formulated. The modified GA operations are purposed to maintain the feasibility of populations. By using the proposed algorithm, one can obtain a task assignment with proper path planning and obstacle avoidance concurrently. Moreover, the simulation results indicate that the proposed genetic algorithm can be used for the complicated group mission with timing constraints. In addition, because the proposed chromosome consists of two levels, a task and a method to accomplish the task, it can be applied to various types of assignment problems of group missions using multiple agents.

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